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ABSTRACT

The calculator is used to investigate properties and characteristics of certain unary operations. These include how a number is related to the number resulting from a unary operation and the result of two unary operations. A discussion of the calculator algorithm to accomplish a series of unary operations is given. Record sheets to summarize discoveries are included. (MP)

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Project Paper 77-5

A MONADIC MODULE alias A UNARY UNIT

by

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Report from the
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Since appreciation is expressed to Leonard Rush (Principal), Ms Jo Marshall(Learning Coordinator), Jack Novick (Teacher) and his seventh-grade mathematics students for their interest and cooperation in making possible an exploratory use of the instructional module or unit within the Thomas Jefferson Middle School (Madison, WI).

This module is excerpted from material prepared for ongoing calculator explorations[1] with a class of accelerated seventh-grade students who will study "first-year algebra" in grade 8 rather than in grade 9. The module is predicated upon several premises, and its use assumes certain prior mathematical and calculator experiences.

Some Premises

1. Unary or monadic operations should receive greater explicit attention than usually is the case in school mathematics programs which put such heavy emphasis upon binary or diadic operations (Hess, 1974).

2. Conventional arithmetic or algebraic notation for unary operators very readily may confuse rather than clarify for school learners the *conceptualization* of unary operations. For instance:

$\log x$	$\sqrt{x}$	or	$x^{\frac{1}{2}}$		
$x!$	x^2				
$\lvert x \rvert$	x^{-1}	or	$\frac{1}{x}$	or	$1/x$
$[x]$	e^x				

As Gilman & Rose (1976) have indicated, "Whatever other merits this mishmash has, consistency certainly isn't one of them, for the symbol which is the functional indicator may appear on the left, the right, both sides, on top, or be in a special position, or be represented by an alphabetical label [p. 57]!"

3. At a pre-algebra level in particular, potential confusion can be minimized and conceptual clarification enhanced by using in connection with all unary functions a consistent *post*-operator or *post*fix notation of the form "a [▽]" that in itself suggests the manner in which such functions are *calculator-executed*: first, operand a is entered or otherwise displayed; then a is "acted upon" by the operation symbolized by operator [▽]. Using this calculator-based notation or language, it then is sensible to

write statements of the form $a \rightarrow a\boxed{\nabla}$ or $a\boxed{\nabla} = b$.

More specifically, the three unary operations that are the focus of this two-part module may be designated by the following operators:

$\boxed{x^2}$: the *squaring* operator;

$\boxed{\surd}$ or $\boxed{\sqrt{x}}$: the *square root* operator; and

$\boxed{1/x}$: the *reciprocal* or *multiplicative inverse* operator.

Although a "mishmash" of *labels* remains embedded in the preceding symbols, the mishmash of *operator* designation can be eliminated:

Conventional notation	Calculator-based notation
$a \rightarrow a^2$ or $a^2 = b$	$a \rightarrow a\boxed{x^2}$ or $a\boxed{x^2} = b$
$a \rightarrow \sqrt{a}$ or $\sqrt{a} = b$	$a \rightarrow a\boxed{\surd}$ or $a\boxed{\surd} = b$
$a \rightarrow \frac{1}{a}$ or $\frac{1}{a} = b$	$a \rightarrow a\boxed{1/x}$ or $a\boxed{1/x} = b$

Two things should be noted at this point.

First, although the calculator-based notation *suggests* in each instance a keystroke sequence to be used when applying a particular unary operation to some number a, this understandably does not make explicit that a "prefix" or "shift" key must be used on some calculators to select the desired unary operation, and that for $a < 0$ a CHS or +/- key must be used along with digit key(s) to enter a.

Second, mention of the CHS or +/- key may raise the following question: Why is not the *additive inverse* operation included among the unary operations in this module? A user of the module certainly may elect to make that inclusion. But it has been my experience with middle-school students at the pre-algebra level that they have *considerable* difficulty distinguishing between use of the CHS or +/- key in connection with data entry and use of the *same* key to perform the additive-inverse *unary operation* upon a number. This is the same problem that students encounter when interpreting an expression such as "-(-5)" in which the same symbol, - ,

is to be viewed in different ways. For this reason I have bypassed the additive-inverse operation in *this* module.

4. Finally, work with unary operations should emphasize properties and relationships. Several of these provide the basis for this module.

Pre-module Background

It is assumed that:

1. Students have worked with *rational* numbers (negative as well as non-negative), and readily can order such numbers in decimal form and can either *truncate* or *round* decimal representations of rational numbers to a specified number of decimal places.

2. Students' work with calculators has included exemplars of each of the following where a is rational and b may be either "exact" or a rational *approximation* (or even *undefined*):

Input	Function key	Output	Interpretation(s)
a	x^2	b	$b = a \times a$
a	$\sqrt{\ }$	b	$b \times b = a$ or $a \div b = b$
a	$1/x$	b	$1 \div a = b$ $\quad b \times a = 1$

The kind(s) of calculator(s) used by students--prior to the module or for the work of the module--is immaterial as far as "logic" is concerned: it may be algebraic, without or with an operational hierarchy, or it may be RPN (reverse Polish notation). The calculators that I have used with the seventh-grade class are RPN-based: specifically, National Semiconductor's "Mathematician" 4510 (and, for demonstration, etc., EduCALC's 29GD which utilizes a Hewlett-Packard 29C programmable).

The Module

The particular relationships and properties to be identified by students may be summarized as follows, where a is a rational number:

Part 1

	#1	#2	#3
Condition	a [x^2] = b	a [$\sqrt{\ }$] = b	a [$1/x$] = b
$a > 1$	$b > a$	$b < a$	$b < a$
$a = 1$	$b = a$	$b = a$	$b = a$
$0 < a < 1$	$b < a$	$b > a$	$b > a$
$a = 0$	$b = a$	$b = a$	Undefined
$-1 < a < 0$	$b > a$	Undefined	$b < a$
$-1 = a$			$b = a$
$-1 > a$			$b > a$

Part 2

	#4	#5	#6
Condition	a [x^2] [$1/x$] = c a [$1/x$] [x^2] = c'	a [$1/x$] [$\sqrt{\ }$] = c a [$\sqrt{\ }$] [$1/x$] = c'	a [$\sqrt{\ }$] [x^2] = c a [x^2] [$\sqrt{\ }$] = c'
$a > 0$	$c' = c$	$c' = c$	$c' = c = a$
$a = 0$	c & c' undefined	c & c' undefined	
$a < 0$	$c' = c$		c undefined (but not c')

#7. a [$1/x$] [$1/x$] = a for all $a \neq 0$.

Teachers may use the appended record sheets (one for each of the seven relationships/properties just identified) in a variety of ways. It is intended, however, that these sheets summarize "evidence" in the form of *specific numerical exemplars generated by students* (subject to teacher suggestion or guidance as needed to provide an adequate input domain) that can lead either to a "discovery" of particular relationships/properties or to a support or denial of *hypothesized* or *conjectured* relationships/properties.

It is suggested that teachers use their own "decision rules" regarding the *recording* of calculator output in truncated or rounded form. It is important, however, that students are aware of the following fact and its potential implications when working with Part 2 questions: The *second* of the two unary operations is applied to the *digits retained internally* by the calculator, which may be more precise than a recorded truncated or rounded value of b or b'. The properties/relationships identified on page 4 of this manuscript apply to rational *numbers* but must be inferred at times from decimal representations that are *approximate* rather than exact.

The record sheets for questions #4, #5 and #6 provide for first recording the steps of a *calculator algorithm*, several of which are illustrated on page 6 of this manuscript. Depending upon the calculators used by students, such algorithms may require that a be input via the keyboard twice rather than just once in connection with each exemplar.

Pages 7 and 8 of this manuscript detail the user instructions and program that pertain to an *EduCALC* GD29 that I have used to advantage when working with students on this module. Once the calculator has been initialized for a particular question and record sheet, the generation of output is facilitated by having to execute only the R/S key, regardless of the unary operation(s) involved.

It is left to the teacher's discretion to decide the extent to which students should be able to *verbalize* the relationships and properties emphasized in this module. In Part 2 in particular, the teacher may or may not wish to make explcit that:

the operators in #4, in #5, and in #6 may be *commuted;*

the operators in #6 are *inverses* of each other; and

the operator in #7 is *its own inverse.*

Calculator Algorithm for Question # 4

Step	Input	Key(s)	Output
1	a	ENT	a
2		F x^2	b
3		$1/x$	c
4		C↓	a
5		$1/x$	b'
6		F x^2	c'

The algorithm above is one of three possibilities suggested by students using the RPN "4510" calculators.

The algorithms at the right are the three algorithms, expressed in a generalized form, that apply to questions 4, 5, 6 when RPN "4510" calculators are used.

Step	Input	Key(s)	Output
1	a	ENT	a
2		∇_1	a ∇_1 = b
3		∇_2	b ∇_2 = c
4		C↓	a
5		∇_2	a ∇_2 = b'
6		∇_1	b ∇_1 = c'

OR

1	a	ENT	a
2		∇_1	b
3		∇_2	c
4		$x \leftrightarrow y$	a
5		∇_2	b'
6		∇_1	c'

OR

1	a	MS	a
2		∇_1	b
3		∇_2	c
4		MR	a
5		∇_2	b'
6		∇_1	c'

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Set display for p places (May be changed at any time)		f FIX p	
	PART 1. Execute steps 2 thru 4.			
2	Initialize for ONE of these: for x^2	1	g RTN R/S	0
	OR for $\sqrt{x}$	2	g RTN R/S	0
	OR for $1/x$	3	g RTN R/S	0
3	Iterate this for each a ▽	a	R/S	a ▽ = b
4	For new ▽, return to step 2.			
	PART 2. Execute steps 5 thru 7.			
5	Initialize for ONE of these permutations:			
	for x^2 $1/x$, $1/x$ x^2	4	g RTN R/S	0
	OR for $\sqrt{x}$ $1/x$, $1/x$ $\sqrt{x}$	8	g RTN R/S	0
	OR for x^2 $\sqrt{x}$, $\sqrt{x}$ x^2	9	g RTN R/S	0
	OR for $1/x$ x^2, x^2 $1/x$	7	g RTN R/S	0
	OR for $1/x$ $\sqrt{x}$, $\sqrt{x}$ $1/x$	5	g RTN R/S	0
	OR for $\sqrt{x}$ x^2, x^2 $\sqrt{x}$	6	g RTN R/S	0
6	Iterate for each a & selected permut.:	a	R/S	a ▽$_1$ = b
			R/S	b ▽$_2$ = c
			R/S	a ▽$_2$ = b'
			R/S	b' ▽$_1$ = c'
7	For new permutation, return to step 5			

If display shows Error at any time, push CLx GTO 1. Then return to step 3 (Part 1) or 6 (Part 2) with a new a.

EduCALC 29GD User Instructions and Part 2 Illustration

NOTE: To generate a ▽ ▽ ....., keystroke a R/S R/S .....

4	g RTN	R/S	0
- 5.12	R/S		26.2144
	R/S		0.0381
	R/S		- 0.1953
	R/S		0.0381

EduCALC 29GD Program

Key(s)	Line	Code
STO 0	01	23 00
CLx	02	34
GTO 0	03	13 00
g LBL 6	04	15 13 06
STO 1	05	23 01
f $\sqrt{x}$	06	14 63
R/S	07	74
g x^2	08	15 63
R/S	09	74
RCL 1	10	24 01
g x^2	11	15 63
R/S	12	74
GTO 2	13	13 02
g LBL 8	14	15 13 08
STO 1	15	23 01
f $\sqrt{x}$	16	14 63
R/S	17	74
g $1/x$	18	15 74
R/S	19	74
RCL 1	20	24 01
g $1/x$	21	15 74
R/S	22	74
g LBL 2	23	15 13 02
f $\sqrt{x}$	24	14 63

Key(s)	Line	Code
GTO 0	25	13 00
g LBL 5	26	15 13 05
STO 1	27	23 01
g $1/x$	28	15 74
R/S	29	74
f $\sqrt{x}$	30	14 63
R/S	31	74
RCL 1	32	24 01
f $\sqrt{x}$	33	14 63
R/S	34	74
GTO 3	35	13 03
g LBL 7	36	15 13 07
STO 1	37	23 01
g $1/x$	38	15 74
R/S	39	74
g x^2	40	15 63
R/S	41	74
RCL 1	42	24 01
g x^2	43	15 63
R/S	44	74
g LBL 3	45	15 13 03
g $1/x$	46	15 74
GTO 0	47	13 00
g LBL 9	48	15 13 09

Key(s)	Line	Code
STO 1	49	23 01
g x^2	50	15 63
R/S	51	74
f $\sqrt{x}$	52	14 63
R/S	53	74
RCL 1	54	24 01
f $\sqrt{x}$	55	14 63
R/S	56	74
GTO 1	57	13 01
g LBL 4	58	15 13 04
STO 1	59	23 01
g x^2	60	15 63
R/S	61	74
g $1/x$	62	15 74
R/S	63	74
RCL 1	64	24 01
g $1/x$	65	15 74
R/S	66	74
g LBL 1	67	15 13 01
g x^2	68	15 63
g LBL 0	69	15 13 00
R/S	70	74
GTO 1	71	13 22

References

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Hess, A. L. Unary operations. *Mathematics Teacher*, 1974, *67*, 281-283.

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Footnote

[1] This material is derived from the writer's fourth year of calculator exploration and investigation, over a range of grade levels, as a Professor of Mathematics Education and Faculty Associate, Wisconsin Research and Development Center for Cognitive Learning--supported in part as a research and development center by funds from the National Institute of Education, Department of Health, Education, and Welfare (Center Contract No. NE-C-00-3-0065). The opinions expressed herein do not necessarily reflect the position or policy of the National Institute of Education and no official endorsement by that agency should be inferred.

#1. How is a $\boxed{x^2}$ related to a ?

Is it the same for every a ?

a	a $\boxed{x^2}$ = b	b ? a
		b a
		b a
		b a
		b a
		b a
		b a
		b a
		b a
		b a
		b a
		b a
		b a

#2. How is a $\boxed{\surd}$ related to a ?
Is it the same for every a ?

a	a $\boxed{\surd}$ = b	b ? a
		b a
		b a
		b a
		b a
		b a
		b a
		b a
		b a
		b a
		b a
		b a
		b a

#3. How is a [1/x] related to a ?

Is it the same for every a ?

a	a [1/x] = b	b ? a
		b a
		b a
		b a
		b a
		b a
		b a
		b a
		b a
		b a
		b a
		b a
		b a

Calculator Algorithm for Question # 4

Step	Input	Key(s)		Output

#4. How does a [1/x] [x^2] compare with a [x^2] [1/x] ? The same for all a ?

a	a [x^2] = b	b [1/x] = c	a [1/x] = b'	b' [x^2] = c'	c' ? c
					c' c
					c' c
					c' c
					c' c
					c' c
					c' c
					c' c

Calculator Algorithm for Question # 5

Step	Input	Key(s)	Output

#5. How does a [√] [1/x] compare with a [1/x] [√] ? The same for all a ?

a	a [1/x] = b	b [√] = c	a [√] = b'	b' [1/x] = c'	c' ? c
					c' c
					c' c
					c' c
					c' c
					c' c
					c' c
					c' c

Calculator Algorithm for Question # 6

Step	Input	Key(s)	Output

#6. How does a [√] [x^2] compare with a ? The same for all a ?

How does a [x^2] [√] compare with a ? The same for all a ?

a	a [√] = b	b [x^2] = c	a [x^2] = b'	b' [√] = c'	c ? a c' ? a
					c a c' a
					c a c' a
					c a c' a
					c a c' a
					c a c' a
					c a c' a
					c a c' a

#7. What is $\boxed{\nabla}$ so that $a\boxed{1/x}\boxed{\nabla} = a$, for $a \neq 0$?

a	$a\boxed{1/x} = b$	$b\boxed{\nabla} = a$
		$b\boxed{} =$
		$b\boxed{} =$
		$b\boxed{} =$
		$b\boxed{} =$
		$b\boxed{} =$
		$b\boxed{} =$
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